Research Memorandum
ETS RM–21-11

Conceptualization and Development of a Performance Task for Assessing and Building Elementary Preservice Teachers' Ability to Facilitate Argumentation-Focused Discussions in Mathematics: The Fractions Between Task

Heather Howell
Jamie N. Mikeska
Jessica Tierney
Benjamin Baehr
Penny Lehman

December 2021

ETS Research Memorandum Series

Conceptualization and Development of a Performance Task for Assessing and Building Elementary Preservice Teachers' Ability to Facilitate Argumentation-Focused Discussions in Mathematics: The Fractions Between Task

Heather Howell, Jamie N. Mikeska, Jessica Tierney, & Benjamin Baehr
ETS, Princeton, New Jersey, United States

Penny Lehman
Consultant

December 2021

Corresponding author: H. Howell, E-mail: hhowell@ets.org

Suggested citation: Howell, H., Mikeska, J. N., Tierney, J., Baehr, B., & Lehman, P. (2021). *Conceptualization and development of a performance task for assessing and building elementary preservice teachers' ability to facilitate argumentation-focused discussions in mathematics: The Fractions Between task* (Research Memorandum No. RM-21-11). ETS.

Find other ETS-published reports by searching the ETS ReSEARCHER database at https://search.ets.org/researcher/

To obtain a copy of an ETS research report, please visit https://ets.org/research/contact/

Action Editor: Laura Hamilton

Reviewer: Gabrielle Cayton-Hodges

Abstract

In this research memorandum, one of a series of eight such reports, we describe the development process by which we produced a series of performance tasks designed for preservice elementary teachers for formative assessment use in the context of teacher education programs. Each performance task provides an opportunity for preservice elementary teachers to practice facilitating an argumentation-focused discussion targeting a student learning goal in elementary mathematics or science. One unique aspect of this work is that the discussions take place within an online simulated classroom environment that consists of five upper elementary student avatars. This report documents the development process at three levels. First, we define the overarching teaching competency that each task targets—the ability to facilitate argumentation-focused discussions—by describing the general approach and processes used to develop the full set of eight tasks and the key components embedded within each task. Next, we describe the academic content addressed in the subset of four mathematics tasks and how the content conceptualization supports the use of the tasks individually or as a set. We then discuss the specific task that is the focus of this research memorandum, outlining how it was designed to capture evidence of the targeted teaching competency.

Keywords: performance task, elementary education, simulated classrooms, virtual reality, discussion, argumentation, preservice teachers, teacher education, mathematics, fractions

Acknowledgments

This study was supported by a grant from the National Science Foundation (Award No. 1621344). The opinions expressed herein are those of the authors and not the funding agency.

The development of these performance tasks, especially the interactor training materials, was enhanced by the feedback from the talented group of Mursion interactors who worked on this project and who served as the human in the loop during these simulated discussions. In addition, we are grateful for the advice and critical review of these tasks from our advisory board members, assessment developers, and research colleagues. Finally, we are appreciative of the teacher educators and preservice teachers who provided substantive feedback on how to improve these tasks for future use in teacher education.

Preface

This research memorandum is one of eight reports in which we describe the development process by which we produced a series of performance tasks designed for preservice elementary teachers for formative assessment use in the context of teacher education programs. The following table provides an overview of the eight performance tasks.

Descriptions of the Eight Performance Tasks

Task name	Task description
	Mathematics
Ordering Fractions	The teacher leads a discussion of three student-generated strategies for ordering a set of given fractions from least to greatest.
Fractions Between	The teacher leads a discussion with the students about an unconventional student-generated method for generating fractions between two given fractions. The discussion is focused on the strengths and weaknesses of the strategy, and its applicability to other situations.
Birdseed	This discussion is grounded in students' work on a story problem in which they have used fraction multiplication. Prior to the discussion, the students individually critiqued one another's work, making the critique aspect of argumentation more clearly available to the teacher.
Eight Divided by One Fourth	This discussion focuses on students' work to generate meaningful understandings and representations of division by a fraction.
	Science
Mystery Powder	This discussion focuses on reaching group consensus around the identity of an unknown powder based on its properties and what is known about a set of common powders. In addition to identifying the mystery powder, students discuss which properties are most useful and why.
Conservation of Matter	In this task, the teacher supports the students in discussing whether the amount of matter is conserved during a physical change, in this case the mixing of ingredients to produce lemonade.
Modeling Matter	This task focuses on critiquing and revising visual models for explaining what happens after a drop of red food coloring is dropped into a cup of water.
Changing Matter	This discussion builds on students' prior work mixing together different combinations of substances and forming claims about whether each combination produced a new substance, with an emphasis on using evidence to support those claims.

Each report is dedicated to a singular task and provides a full description and corresponding appendix text for that particular task. All of the reports include a description of the general development process that applies to the full set of tasks. Additional materials to support the use of the performance tasks, such as interactor training and scoring documentation, are not included in these reports but are archived and publicly available through the Qualitative Data Repository housed at Syracuse University (https://data.qdr.syr.edu/dataverse/go-discuss).

The first section of this report details the development of the performance tasks, including a description of the construct, the task type, and the process used to develop each of the eight tasks. In the second section, we discuss the content focus of the set of mathematics tasks and of the Fractions Between task in particular. In the final section, we describe the resulting set of materials that make up the stable components of the task itself and use examples from the Fractions Between task to illustrate what these components look like and how they function together in the performance task.

Section 1: Development of the Performance Tasks

In this section of the report, we share our conceptualization of the teaching practice of facilitating argumentation-focused discussions, describe what a simulated teaching performance task is, and explain how our use of the performance task maps onto the conceptualization of the teaching practice. We finish by outlining the process steps that we used to develop the tasks.

Construct Definition: Facilitating Argumentation-Focused Discussions

Our construct of interest is the teaching practice of facilitating discussions that engage students in argumentation, or what we refer to as "facilitating argumentation-focused discussions." We focused on this teaching practice for a number of reasons. First, facilitating argumentation-focused discussions is an ambitious teaching practice that is critically important for teachers to learn how to do well in order to support student conceptual learning within content areas (Kazemi & Stipek, 2009; Russell et al., 2017; Stylianides et al., 2016; Walshaw & Anthony, 2008). Second, this practice is hard to learn how to do well, and many teachers—even experienced teachers—tend to have had little opportunity to learn how to do well (Barkai et al., 2002; Reid & Zack, 2009). Finally, the focus on argumentation was purposeful. Although teachers may facilitate many kinds of discussions with K–12 students, both the *Common Core State Standards* and *Next Generation Science Standards* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Research Council, 2013) identify argumentation as one of the key mathematical and scientific practices that K–12 students need to master.

To define the construct that we were aiming to measure—preservice elementary teachers' ability to facilitate argumentation-focused discussions—we began by reviewing the empirical and practitioner literature as well as the current student standards in mathematics and science to identify the core aspects of this teaching practice. Building on this review, we identified five dimensions of high-quality, argumentation-focused discussions: (a) attending to students' ideas, (b) developing a coherent and connected storyline, (c) encouraging student-to-student interactions, (d) developing students' conceptual understanding, and (e) engaging students in argumentation. Table 1 provides details about the specific focus of each dimension.

Table 1. Dimensions of a Scoring Rubric to Evaluate Preservice Teachers' Ability to Facilitate Argumentation-Focused Discussions

Dimension	Description: Degree to which the teacher. . .
Attending to students' ideas	. . . is being responsive to students, with a focus on making sure the discussion is grounded in the ideas the students bring with them
Developing a coherent and connected storyline	. . . is able to shape a coherent discussion, with a focus on building and connecting ideas toward an instructional goal
Encouraging student-to-student interactions	. . . organizes the classroom community and the social interactions so students respond directly to one another's thinking
Developing students' conceptual understanding	. . . makes productive decisions about how to address particular ideas, especially students' misunderstandings
Engaging students in argumentation	. . . emphasizes disciplinary argumentation (e.g., consideration of opposing claims; facilitates critique and rebuttals; encourages students to draw upon evidence and reasoning)

Note. Adapted from "Using Performance Tasks Within Simulated Environments to Assess Teachers' Ability to Engage in Coordinated, Accumulated, and Dynamic (CAD) Competencies" by J. N. Mikeska, H. Howell, and C. Straub, 2019, *International Journal of Testing*, *19*(2), p. 138 (https://doi.org/10.1080/15305058.2018.1551223). Copyright 2019 by Taylor & Francis.

The first dimension, attending to students' ideas, focuses on the extent to which teachers are responsive to students' ideas in equitable ways, ensuring that the discussion is grounded in students' ideas and that all students are engaged in meaningful aspects of the discussion. The second dimension, developing a coherent and connected storyline, targets the degree to which the teacher can shape a coherent discussion by building and connecting ideas toward a learning goal. The third dimension, encouraging student-to-student interactions, pays attention to how teachers facilitate the discussion so students are the ones responsible for interacting directly with each other and engaging with one another's ideas. The fourth dimension, developing students' conceptual understanding, targets the extent to which the teacher and students are involved in evaluating the accuracy and validity of key ideas and how well the teacher productively addresses students' misunderstandings. The fifth dimension, engaging students in argumentation, emphasizes the degree to which students are invited to and engage in argument construction and critique during the discussion.

Simulated Teaching Performance Tasks

The overall goal of our research was to develop a set of simulation-based performance tasks that could be used to assess and build preservice elementary teachers' ability to facilitate argumentation-focused discussions. We conducted this work in the context of an innovative, mixed reality platform (see Figure 1)—an upper elementary simulated classroom composed of five student avatars.

Figure 1. Image of an Upper Elementary Simulated Classroom

Credit: Image courtesy of Mursion

The student avatars are controlled on the back end by a human in the loop, called an *interactor*, who is trained to respond as each of the five student avatars during the discussion. The preservice teacher does not see the interactor but instead views the student avatars on a television or computer screen and can interact with the student avatars in real time during the discussion. We hypothesized that the simulated classroom could serve as a practice-based space for preservice teachers to hone their skill in this teaching competency. Each performance task was designed to be deployed within the upper elementary classroom environment.

The teaching competency of facilitating argumentation-focused discussions is one that involves complex interactions between a teacher and students around specified content. It requires a practice space that provides opportunities for extended interactions to unfold over time, as a teacher's ability to engage in this practice is observable only across these patterns of interactions (Mikeska et al., 2019). In earlier writing, we describe this competency as one that is "coordinated," "accumulated," and "dynamic" (Mikeska et al., 2019, pp. 132–133). By *coordinated*, we mean that the teacher is required to manage multiple, sometimes competing, considerations simultaneously—for example, trying to balance the goal of engaging students in argumentation with addressing students' erroneous conceptual understanding. *Accumulated* refers to the nature of the evidence that needs to be captured, as the teaching competency is observed over time across the patterns of interactions and not by examining individual, disparate interactions. By *dynamic*, we mean that this teaching competency is observed as teachers respond to the constantly changing nature of various task conditions. Each one of these aspects has implications for task design.

First, to ensure that we were adequately measuring this teaching competency, we had to ensure that our task design afforded teachers the opportunity to manage various considerations at the same time. Second, we had to ensure that the tasks provided substantial opportunities to capture evidence at various time points. For example, the tasks needed to provide us opportunity to observe how teachers prompt (or fail to prompt) direct student dialogue and the ways that students begin to engage in specific behaviors more (or less) frequently based on this teacher prompting over time. Finally, we had to create variable task situations so that the teacher would be required to respond to the changing nature of the situation over time, for example, creating dynamic student profiles where students can “learn” based on their interactions with other students and the teacher, as described in the final section of this report. In the next section, we explain our process for developing each performance task, which includes both the preservice teacher–facing task materials and the interactor-facing task materials.

Overview of the Task Design Process

Because the overall goal of using these tasks was to be able to make valid inferences about preservice teachers’ ability to facilitate discussions that engage students in the practice of argumentation, we drew upon the process of evidence-centered design (Mislevy et al., 2002) to develop our evidence model. We then used this evidence model to inform the overall design of each performance task. Our first step was to use our construct definition to develop an evidence model to articulate the observable behaviors that could serve as evidence of preservice teachers’ ability to engage successfully in each dimension. For example, for the first dimension—attending to students’ ideas—we identified three indicators of that dimension of this overall teaching practice, including the preservice teachers’ abilities to incorporate ideas from the students’ written prework into the discussion, to elicit substantive ideas from all students, and to make use of students’ ideas to move the lesson forward in regard to the discussion’s specified student learning goal. We further elaborated each one at three levels of proficiency—beginning novice, developing novice, and well-prepared novice—to describe the observable behaviors one would gather evidence about to inform assessment of that indicator. For example, for the previously discussed dimension, attending to students’ ideas, under the second indicator, eliciting substantive ideas, the observable behaviors specified (Figure 2) indicate that elicitation of substantive ideas from students is related both to the teacher’s sustained efforts to elicit such contributions and to the teacher’s success in eliciting such contributions from all students. Substantive ideas are defined as those that go beyond yes/no statements or restatements of the work the student completed before the discussion.

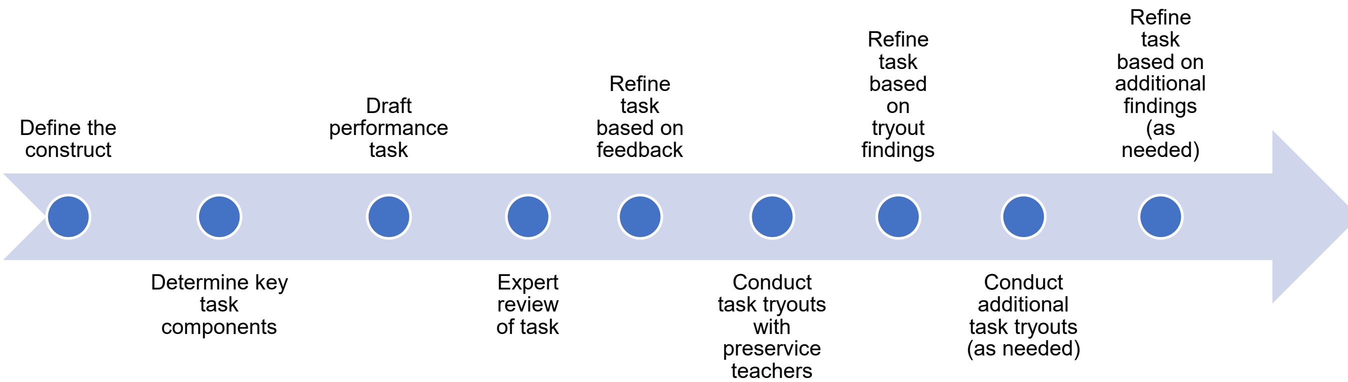
Figure 2. Example of Observable Behaviors for Indicator 1b: Elicits Substantive Contributions

Indicator	Level 1 Beginning Novice	Level 2 Developing Novice	Level 3 Well-Prepared Novice
1b. **Elicits Substantive Contributions**	The teacher does not probe students for substantive contributions or does so only once or twice. OR The teacher interacts with only one student from each group.	The teacher probes students for substantive contributions intermittently during the lesson. AND The teacher does not elicit a substantive contribution from at least one student.	The teacher probes students for substantive contributions consistently throughout the lesson. AND The teacher succeeds in eliciting one or more substantive contribution from **every** student.

Our project's advisory board, made up of teacher educators, content specialists, and researchers in mathematics and science teacher education, conducted an expert review of these dimensions and indicators. Their goal was, first, to ensure that they were adequately aligned to the construct and previous literature in mathematics and science teacher education and, second, to provide feedback on whether our characterization of high-quality discussions in the context of disciplinary argumentation adequately addressed the ways in which this teaching practice is used, valued, and characterized within each of the disciplines (elementary mathematics and science). The advisory board also identified and offered suggestions for any aspects of our construct definition that were missing, misrepresented, or not sufficiently addressed. Finally, they considered whether the progressions seemed logical, comprehensive, and scoreable and captured the most important observable teacher behaviors for each dimension and indicator.

In the design of the performance tasks themselves, we used a design-based research approach (Design-Based Research Collective, 2003) in developing and refining the task materials[1] at multiple stages and leveraging various expertise from teacher educators, researchers, preservice and in-service teachers, content experts, and assessment developers (Figure 3). After defining the construct of interest, including the specific dimensions and indicators of this teaching practice, the next step in our task design process was to determine the key task components that would provide opportunities for the preservice teachers to engage in these dimensions of this teaching practice and support us in capturing adequate evidence across all five dimensions.

Figure 3. Progression of the Design-Based Research Process



The task materials include two types of components: the preservice teacher–facing and the interactor training materials. The preservice teacher–facing materials include a written document that provides information to the preservice teacher about the simulated discussion's student learning goal, where this discussion fits into a larger instructional sequence, and what the instructional activities are that the student avatars engaged in prior to this discussion. This document also shows the preservice teacher written work samples that the student avatars generated prior to the discussion, which provides the preservice teachers with insight into the students' sense-making about the specific mathematics problem or science investigation that is the focus of the discussion. In addition, we developed materials to train the interactor. These training materials are designed to help the interactor learn about the student avatars' initial ideas and understandings related to the mathematics problem or science investigation that is the focus for the discussion. These materials also support the interactor in learning about the circumstances under which the student avatars can arrive at new understandings based on ideas and arguments that the preservice teacher or the other students make during the discussion. This level of training is also critical in helping to support the standardization of opportunity across preservice teachers so that experiences in the simulated classroom are comparable in the level and nature of challenge each preservice teacher encounters (Howell & Mikeska, 2021). We describe specific components for these task materials, including how we designed them to capture adequate evidence of the five dimensions of this teaching competency, in the final section of this manuscript.

The next step in our task development process included an expert review of the performance task materials to ensure that the task components (both preservice teacher and interactor facing) worked synergistically to gather observable evidence of the preservice teachers' ability to facilitate argumentation-focused discussions. Reviewers included our advisory board members and assessment development experts, who reviewed the preservice teacher–facing materials to ensure that (a) the task provided opportunities for the preservice teachers to engage the student avatars in the practice of mathematical or scientific argumentation; (b) the written student work samples captured a range of typical responses for upper elementary students regarding the specific mathematics problem or science investigation that was the focus of the discussion; and (c) the educative features in the task would be useful to support preservice elementary teachers in learning how to facilitate high-quality, argumentation-focused discussions centered around these student learning goals. Reviewers also considered whether each of the preservice teacher–facing task components—such as the student learning goal, specific instructional scenario, task description, and student profiles—were clear, appropriate, and sufficient for the intended audience. In terms of the interactor training materials, reviewers focused on ensuring that we identified reasonable responses for the interactor to use as the discussion unfolds in the simulated classroom and that the responses did not limit or misrepresent the preservice teachers' ability to engage in this teaching practice. Our research team then revised these task materials based on the experts' feedback. These revisions included a variety of different changes across these tasks, such as more clearly articulating the discussion's student learning goal, modifying the written student responses to better align with grade-level expectations, refining the teaching tips to provide more robust educative supports for the preservice teachers, and updating the lesson overview and background sections to ensure that the preservice teachers understood where they were being dropped into a larger instructional sequence.

Once we had developed task materials that we hypothesized would allow us to make valid inferences about preservice teachers' ability to facilitate argumentation-focused discussions, we then engaged in a set of tryouts for each performance task. For each tryout, we recruited five to 10 preservice teachers to pilot the task with us within the simulated classroom.

Prior to each tryout, our research team trained interactors on how to enact the student avatars' responses in alignment with the student thinking profiles developed for each task. During the interactor training, we systematically gathered additional information to inform revisions of those materials, assigning a team member as a dedicated observer for each section of training to document where the interactors needed additional support.

For the tryouts, each participating preservice teacher reviewed the preservice teacher–facing task document to prepare for their simulated discussion and then facilitated a discussion for up to 20 minutes with the five student avatars in the simulated classroom. Our research team video recorded these discussions and later scored each one based on the scoring rubric we had developed from the five dimensions of our construct and the progression levels for each dimension. We also gathered self-reported data from each preservice teacher via a task survey and semistructured interview to learn about their perceptions of the task authenticity, interactions with the student avatars, their discussion performance, and the usefulness of the simulated teaching experience integrated within mathematics and science elementary method courses. Our research team analyzed these data sources to identify patterns in the preservice teachers' perceptions of these task materials and their discussion performances and then used the tryout findings to refine the task materials and our scoring rubric further.

These revisions, like those that took place after expert review, included attention to the clarity of wording throughout the preservice teacher–facing materials, which involved simplifying wording and presentation, refining the teaching tips to call attention to points that had been misunderstood, and in a few cases, revising the core content of the task to better fit the 20-minute time limit. We also revised interactor training materials to provide more support in areas that we had observed to be difficult and to refine language where we had observed it to be confusing to one or more interactors during training. For four of the eight tasks, the resulting revisions were substantial enough to warrant a second round of tryouts using a similar process of data collection, analysis, and task refinement.

Once we finalized the task materials and scoring rubric based on the tryout findings, we then used them in the research project's main study within multiple sections of elementary mathematics and science courses at three different universities in the United States.

Section 2: Content Focus of the Performance Tasks

Each set of performance tasks in mathematics or science is grounded in a single *high-leverage content area*, which, as described in Martin-Raugh et al. (2016), is operationalized following the model of Ball and Forzani's (2011) high-leverage practice framework to include content of the student curriculum that is foundational, spans multiple grade levels, and makes up a significant component of the student curriculum and in which students often struggle absent strong instruction. In other words, it is the content that is most consequential for students to learn well and, therefore, most important for teachers to teach skillfully. In science, the content area of focus is matter and its interactions; in mathematics, the content area of focus is fractions and operations with fractions.

Content Focus of the Math Tasks

In 2016, Martin-Raugh et al. identified fractions and operations with fractions as one of the high-leverage content topics within the elementary mathematics curriculum via a systematic analysis of the mathematics content of the Common Core State Standards. Research has widely acknowledged that fractions and operations on fractions is a difficult content area for teachers to learn how to teach; however, there exists a broad empirical research base on which to model common student understandings and misunderstandings (Ball, 1993; Lamon, 2012; Newton, 2008).

One goal of this focus on high-leverage content was to create a coherent and connected set of performance tasks that would fit together across the time span of a semester, make sense in sequence, and include core content that teacher educators likely would have made a focus of instruction in their work with preservice elementary teachers. Within the set of mathematics performance tasks, the Fractions Between, Birdseed, and Eight Divided by One Fourth tasks were designed to be presented in order across the semester, as this was our envisioned use case, while still standing alone if later used individually outside of a sequence. Fractions Between focuses on a student-generated method for identifying fractions between two given fractions and deciding whether that method is valid and generalizable. The Birdseed task is organized around student solutions to a given word problem for which an area model is used to represent the multiplication of fractions in finding a solution. The Eight Divided by One

Fourth task is grounded in the question of what it means to divide by a fraction and presupposes that it is the student avatars' first exposure to fraction division.

This ordering of ideas met two prespecified criteria. First, this sequence allows the student avatars to appear to advance through a typical mathematics instructional sequence by moving from a focus on considering strategies for identifying fractions between two given fractions to then considering various operations (multiplying and then dividing) with fractions. Second, the specific mathematics of each task does not depend directly on the mathematics of the prior task, minimizing instances where a preservice teacher might expect a particular student avatar to remember the exact content of the prior task. The Ordering Fractions performance task, unlike the others, was designed to be used as a pre and post measure at the beginning and end of an elementary methods course and therefore needed to be conceptualized such that it would be reasonable for preservice teachers to engage in before and after the other three tasks. Fraction comparison is a topic that teachers often return to at different points of the curriculum and in which students can engage with different levels of sophistication across multiple grades, making it a good fit to this purpose. The Ordering Fractions, Birdseed, and Eight Divided by One Fourth tasks are described in detail in other reports in this series. In the following section we describe in more detail the Fractions Between task, which is the focus of this report.

The Fractions Between Mathematics Task

The Fractions Between performance task is focused on how students make sense of and evaluate the viability and generalizability of an unconventional student approach to generating fractions between two given fractions, 2/3 and 7/8. The preservice teacher is provided a packet of materials (the preservice teacher–facing materials) prior to facilitating the discussion in the simulated classroom following a template used across the full set of tasks. The packet describes the "one less" method created by Emily, one of the students, in which she noticed that the numerators of 2/3 and 7/8 are each one less than the denominators. She generated a list of other fractions (3/4, 4/5, 5/6, 6/7), reasoning that, if lined up correctly, they must fall between the given fractions. The five students, working in two small groups, have already been asked to react to Emily's proposed method. The packet includes written work in which they have responded to a series of questions about whether the method works, whether it would work for other pairs of fractions, and whether they feel it is worth keeping in their toolbox. On the first point, both groups note that the fractions identified by the method do, in fact, fall between 2/3 and 7/8. However, one group (Jayla, Carlos, and Emily) conclude that the method works where the other is unconvinced, pointing out that there are other fractions between 2/3 and 7/8 the method fails to identify. On the second point, both groups agree about some examples for which it would not work, but the examples are not fully explained. They also disagree about whether the method is worth keeping in the toolbox. This intentional variation across the two groups provides opportunities for the preservice teacher to guide the students in comparing their answers, determining which examples are relevant and why, and also guiding them toward consensus around their claims that the method works or generalizes with attention to whether the supporting reasoning is valid.

Section 3: The Generalized Task Design

As referenced previously, one of the outcomes of the design-based research process described in the first section was the development of a stable set of task components to be used across all eight (four science and four mathematics) tasks and designed to support a consistent experience for preservice teachers. The resulting template can be used to support future development by providing a structure for newly developed tasks with different content and is described here in the context of the Fractions Between task.

Each task is made up of two types of components: the preservice teacher–facing materials and the interactor training materials. Table 2 lists the task components of the preservice teacher–facing materials, which includes three documents for each task. The Introduction to the Simulated Classroom and the Warm-Up Task are separate handouts that are used in common across all eight performance tasks and provide an overview of how the simulation works and a brief familiarization exercise to get the preservice teacher started before they lead the discussion. The main document is the performance task itself (see the appendix for the full text of the Fractions Between performance task), which is designed to help the preservice teacher plan for and lead the discussion. Task components in the preservice teacher–facing task document include the sections Introduction to the Task, Lesson Overview, Student Responses, Making Sense of the Student Work, Shared Workspace Pages, Features of High-Quality Discussions Focused on Argumentation, and Video Examples of High-Quality Discussions Focused on Argumentation. Teaching tips appear throughout the document rather than as a separate section.

Table 2. Components of the Preservice Teacher–Facing Task Materials

Component	Purpose
Introduction to the Simulated Classroom (separate handout)	This stand-alone handout acquaints the preservice teacher with the basic functionality of the simulated classroom as well as introduces them to each of the five students via short bios. It also includes links to short videos in which the students introduce themselves.
The Warm-Up Task: Taking the Students' Lunch Orders (separate handout)	The warm-up task, which takes about 5 minutes, is a scripted task in which the preservice teacher takes the students' lunch orders. It is intended to allow the preservice teacher to become accustomed with the simulated environment before starting the discussion.
Introduction to the Task	This task component orients the preservice teacher to the task. It includes a clear statement of the student learning goal and what the preservice teachers should aim to do during the discussion.
Lesson Overview	This task component situates the 20-minute discussion within the larger lesson and instructional sequence, describing students' background knowledge as well as what transpired in the class before the discussion began.
Student Responses	This task component provides each student group's written work, which was generated prior to the discussion.
Making Sense of the Student Work	This task component complements the student responses and provides explanatory text to help the preservice teacher understand the students' written work. The explanatory text identifies salient features of the students' ideas that might inform the discussion.
Shared Workspace Pages	This task component includes copies of the written student work and any other relevant reference material (e.g., class data table). It can be printed out for use during the discussion.

Component	Purpose
Features of High-Quality Discussions Focused on Argumentation	This task component is a short list of the key features of high-quality discussions as we have defined them, and includes a set of questions about each feature. The preservice teacher can use the questions before or after the discussion to support them in considering how well their discussion will or did meet the task's specified student learning goal.
Video Examples of High-Quality Discussions Focused on Argumentation	This task component provides links to publicly available examples of classroom discussions that illustrate some of the features of high-quality argumentation-focused discussions. The preservice teacher can use the examples to better understand these features and how to incorporate them into their discussion.
Teaching Tips	This task component is embedded throughout the preservice teacher–facing materials and includes teaching tip bubbles that call attention to important ideas about how the discussion might be planned and enacted.

Table 3 lists the components of the interactor training materials, including a series of lessons that combine self-study modules with planned interactive practice with a content expert or trainer in order to help the interactor master the delivery of the task in the simulated classroom.

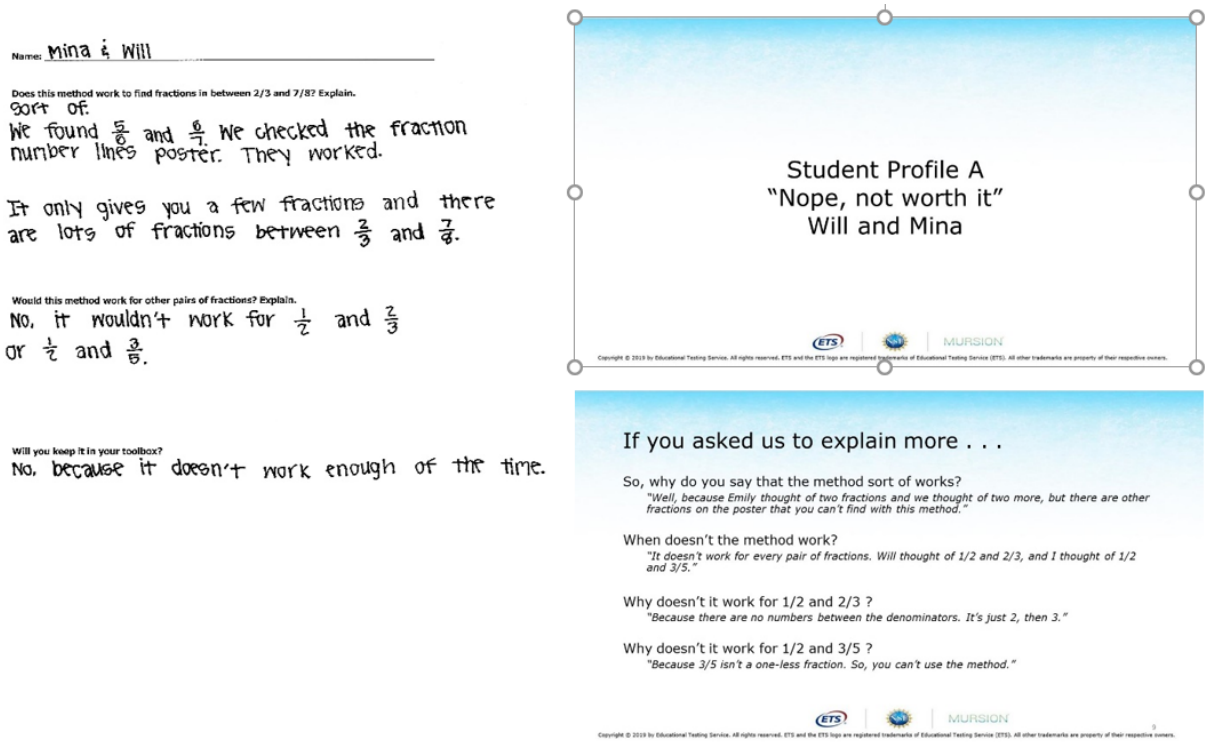
Table 3. Components of the Interactor Training Materials

Component	Purpose
Non-Task-Specific Training	The non-task-specific training materials cover the discussion construct, direct interactors in how to be responsive to teacher prompts to engage in student-to-student interaction, and include the "testing the waters" guidelines. This component also includes independent study of the warm-up activity materials and culminates with an interactive practice session between the interactor and a trainer.
Task-Specific Lessons 1 & 2	For each task, Lesson 1 is an overview of the task and Lesson 2 is an overview of the student profiles for that task, including independent video-guided study of what each student thinks initially as well as how their thinking may shift over time.

Component	Purpose
Task-Specific Lesson 3: The Student Profile Check Out	Lesson 3 is a face-to-face session in which the trainer leads the interactor through a standardized set of questions to ensure adequate mastery of the student profiles for the task.
Task-Specific Lesson 4: The Observational Workshop	For Lesson 4, the interactor meets with two trainers, one of whom plays the part of a teacher and enacts four separate practice discussions while the second trainer provides targeted feedback on the interactor's performance. The four teacher profiles are carefully constructed to represent the breadth of discussion approaches the interactor is likely to encounter.
Task-Specific Lesson 5: The Final Check Out	Lesson 5 is also a face-to-face session with a trainer who enacts two more teacher profiles. Recordings of the session are uploaded and scored by the trainer for adequate fidelity to interactor training guidelines.

The content of the preservice teacher and interactor components are deeply intertwined. For example, a core part of the preservice teacher–facing materials is the presentation of student work that the student avatars have completed in advance of the simulated discussion (component: Student Responses). Every task includes this component, although the number of student groups varies. For each task, then, this necessitates a parallel component of the interactor training (component: Task-Specific Lessons 1 & 2) in which the interactor learns the student avatars' initial ideas and the work they have done prior to the discussion as well as their dynamic content profiles that dictate how their understandings would change over time in response to the teacher's (or other student avatars') statements or questions. For example, for the Fractions Between task, the written student work clearly indicates that Mina and Will do not think the method would always work. They give two examples of fractions pairs for which it would not work, without explaining why. It is up to the teacher to decide how to elicit their thinking. The accompanying interactor training specifies that, if prompted, they would explain that it would not work for fractions like 1/2 and 2/3 because their denominators are adjacent, and thus there is not a whole number between to serve as denominator. It would also not work for fractions such as 1/2 and 3/5 because 3/5 is not a one-less fraction (Figure 4).

Figure 4. Linked Components of Materials Include Student Work (for the Preservice Teacher) and Instructional Videos (for the Interactor)



All components were designed, to the greatest extent possible, to be uniform in ways that are adaptation-friendly, allowing for the insertion of new content as needed to create new tasks. We next discuss some of the critical design considerations that informed our design of the task components using specifics from the Fractions Between task to illustrate how some of these considerations are taken up and addressed in this performance task.

Design Consideration: Knowing Where to Start

The stand-alone Introduction to the Simulated Classroom as well as the Introduction to the Task and Lesson Overview components of the preservice teacher–facing materials are collectively intended to support the preservice teacher in knowing how to begin the discussion. In early tryouts, we realized that one of the logistical elements of the simulation we needed to manage was launching the preservice teacher straight into the discussion, as each teacher has only 20 minutes of simulation time and needs to use it for the intended interactive work of facilitating a discussion. A preservice teacher who spends time doing something else might well use up the full 20 minutes without engaging in the intended content discussion. For example, a natural starting point for preservice teachers encountering new students is to review prior knowledge, but reviewing what they already know takes time away from addressing the student learning goal in the task during the discussion. The Introduction to the Simulated Classroom, Introduction to the Task, and Lesson Overview components acquaint the preservice teacher with the students' prior knowledge and describe exactly what has come before the discussion so that the preservice teacher has a clear sense of where they are to begin the discussion.

Design Consideration: Understanding the Task Purpose

One area in which we found it necessary to build in substantial support across task components was helping the preservice teachers understand that they should be encouraging students to interact with one another directly. Many novice teachers struggle to engage students in this way. In contrast, a frequent and less productive pattern of engagement is known as the initiate—respond—evaluate (IRE) response pattern (Cazden, 1988) in which the teacher interacts with individual students in turn, intervening at each step. One goal of these performance tasks is to support preservice teachers in learning to avoid this pattern. However, if the preservice teachers interpret the instruction to "facilitate a discussion" as asking them to engage in IRE, they may not realize that they are not attempting to meet the intended goal. Paired with a technology environment in which a preservice teacher may not realize the students can speak to one another directly, there is some risk of misdirection on the preservice teacher's part. That misdirection would have represented a source of measurement error for us as it would be difficult to distinguish performances in which the preservice teacher was unable to elicit student interaction from those in which the preservice teacher did not understand that student interaction was possible.

We sought to counter this challenge in several ways across components of both the preservice teacher–facing and interactor training materials. First, we clarified the discussion goal across all tasks to make it clear that student-to-student interaction was possible and desirable. For example, Figure 5 shows text from the Introduction to the Task component stating, "You can encourage the students to talk to one another, ask one another questions, and respond to one another's ideas." Along with this instruction is a teaching tip bubble that cautions the preservice teacher to allow wait time for students to respond.

Figure 5. Text From the Introduction to the Task Component Supporting Student-to-Student Dialogue

TEACHING TIP:
The student avatars are not computer programmed, so it may take them a little time to think and respond.

You will have up to 20 minutes to lead this discussion in a simulated classroom environment made up of five upper-elementary student avatars. The students will be able to hear and see you, and they will respond in real time just like students in a real classroom. You can encourage the students to talk to one another, ask one another questions, and respond to one another's ideas.

Depending on how the discussion unfolds, you may or may not reach a satisfying conclusion by the end of the session time, and it is fine if you do not. If you do not reach a satisfying conclusion, just wrap up the discussion and indicate that you will pick the discussion up during the next class.

Then, on the interactor side, we built in two deliberate instances of student-to-student dialogue intended to help make sure the preservice teacher is aware that direct student interaction is possible (both of these instances are addressed as part of non-task-specific training as they are common across all eight tasks). First, during the warm-up task, one student jumps in and speaks directly to another student. Second, at some point during the first few minutes of the discussion, the interactor is instructed to engage in what we call "testing the waters," by having one student jump in and engage in a brief back-and-forth dialogue with another. In general, the interactor will not have the students engage in this way without prompting, as the preservice teacher is supposed to be learning how to elicit such interaction. But for testing the waters, the interactor makes an exception. This dialogue serves two purposes: First, it is an additional reinforcement to the preservice teacher that students can speak directly with one another, and second, it gives the interactor valuable information about the preservice teacher's initial stance toward how student centered they would like the discussion to be. If the preservice teacher tries to quiet the students or asks them to raise hands, these are signs that the preservice teacher may be discouraging direct student-to-student interaction. However, if the preservice teacher encourages or praises the students or tries to build on the interaction, these are signs that the preservice teacher may be encouraging it.

Design Consideration: Support in Unpacking Student Thinking

Each task is designed on the premise that students have already worked on a given problem in advance of being called together to discuss their work. The students' written work is provided ahead of time (task component: Student Responses) so that the preservice teacher can review and plan the discussion based on it. In addition, we provide information to the preservice teacher to help them make sense of the written student work (task component: Making Sense of the Student Work). This text specifies, for example, whether the answer the students have given is correct, partially correct, or incorrect, what they might have been thinking about, and calls attention to important things the preservice teacher might notice or pay attention to in planning the discussion. For example, for Mina and Will's work discussed previously, as to the question of whether the method works, this text states that their answer to the first question is technically incorrect. The text points the teacher to the fact that, in pointing out that other fractions exist between 2/3 and 7/8, the students are taking up a mathematically interesting question but are not addressing the relationship between this statement and the claim that the method works (Figure 6). The text then states that they are correct that the method does not work for all fractions, points out a similarity between their work and that of the other group, and notes that their explanation of the examples is unclear. It also notes that the evidence and reasoning they use to support their argument that the method is not worthwhile is sound. Noticing the first point opens an avenue for argumentation by pointing the teacher to how the students have invoked incorrect reasoning to support their first claim, insufficient reasoning to support their second claim, and sound reasoning to support their final claim.

Figure 6. Making Sense of the Student Work Component for Mina and Will

Making Sense of Mina and Will's Work

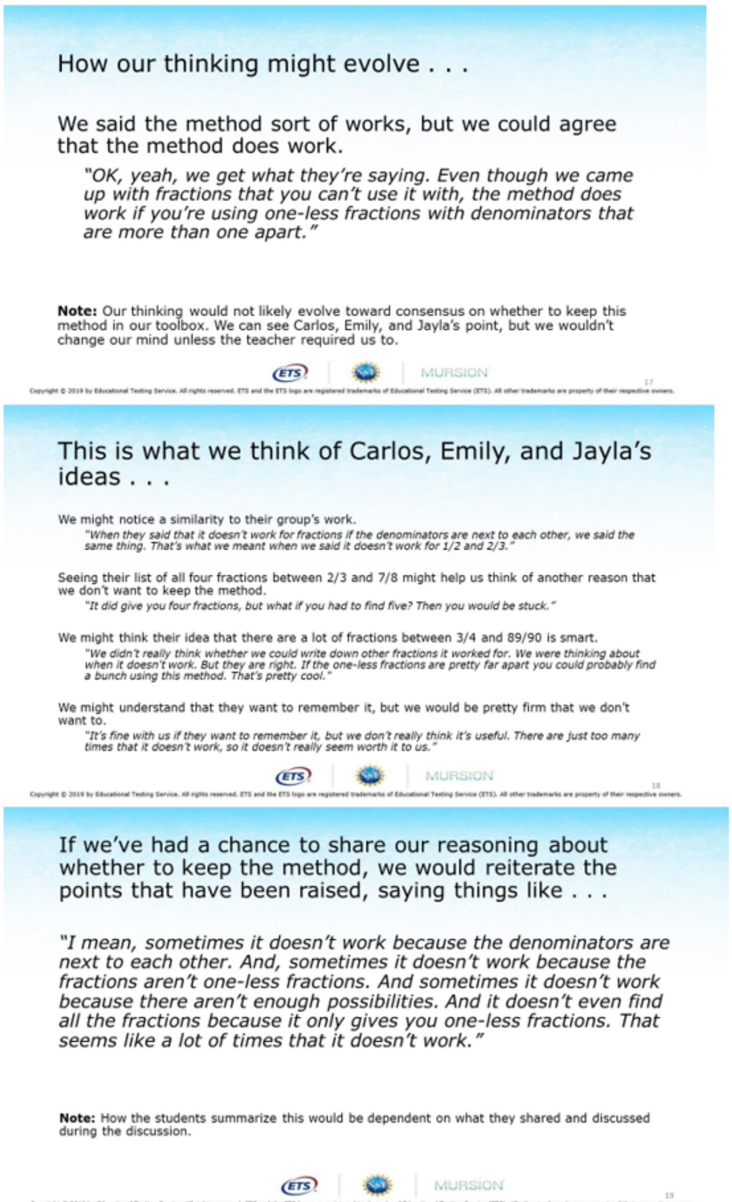
Things to notice about Mina and Will's ideas about the method:

- The answer "sort of" is technically incorrect. The method does work to find fractions in between 2/3 and 7/8.
    - The question this group seems to be answering by writing "sort of" is "does the method give any fractions between 2/3 and 7/8?" This was not the question that was asked, but it is a reasonable question to consider when building their argument.
- They are correct that both 5/6 and 6/7 are between 2/3 and 7/8.
- They are also correct that there are many more fractions that are between 2/3 and 7/8 that the method does not generate. For example, 5/7 and 7/10 are between 2/3 and 7/8, but the method does not produce these fractions.
- They are correct that this method does not work for every pair of fractions.
    - The method does not work for their first example, 1/2 and 2/3, because there are no denominators between 2 and 3. The other group makes a similar statement when they write that the method "wouldn't work for two fractions if the denominators are next to each other."
    - The method does not work for their second example, 1/2 and 3/5, because the fractions are not both one-less fractions.
    - They did not explain why the method fails to work in each of these two examples.
- The evidence and reasoning that they use to support the argument against keeping the method in the toolbox is valid. There are significant limitations on the pairs of fractions for which the method can be used.

Design Consideration: Static and Dynamic Student Profiles

As mentioned previously, interactor training includes both static profiles for students' personalities and initial content ideas as well as dynamic profiles reflecting their likely patterns of change. One characteristic of these tasks is that the students will contribute most of the key ideas if the preservice teacher is facilitating the discussion in a productive way. This means that interactors need training in both when to introduce those ideas and how the individual student avatars should respond to those ideas or sets of ideas, whether presented by the preservice teacher or by other students in response to the preservice teacher's prompting. In the Fractions Between task, for example, Mina and Will's first answer is not correct. The interactor training (Figure 7) notes specific ways in which their thinking would shift on this point, including noting that they could be convinced that the method does work, ways they would react to some of the ideas present in the other group's work, and the fact that their opinion that it just doesn't work enough to justify keeping it in the toolbox is unlikely to change regardless. The ideas that shift their thinking could come from classmates, the teacher, or a combination.

Figure 7. Slides From the Interactor Training: How Mina and Will's Thinking Might Evolve



Conclusion

The preservice teacher–facing materials for the Fractions Between task can be found in the appendix. Our goal in this project was to create a set of simulation-based performance tasks that can be used to support preservice teachers in learning how to facilitate argumentation-focused discussions in two content areas: mathematics and science. In that project work, we video recorded each preservice teacher's discussion session for each task and provided detailed written feedback as well as access to the video to both the preservice teacher and the course teacher educator. We hypothesized that that preservice teacher would be supported on multiple levels. First, there is an aspect of experiential learning, as the preservice teacher sees the student avatars engage in response to their prompts during the simulation. Second, the preservice teacher learns from the written feedback. Although we provided feedback to the preservice teachers, that feedback could also come from a teacher educator or coach, or the preservice teacher could be guided in self-reflection. Third, the performance tasks provide a type of formative assessment information to the teacher educator who can see, in looking across the videos or the feedback, patterns in class or individual performance that allow the teacher educator to adjust instruction within the methods course.

Our design process was deliberately systematic and was intended to support productive adaptation of the task materials that resulted. Although our work took place in the context of preservice teacher learning and for use with the Mursion simulated classroom environment, the tasks could easily be used for professional development and adapted for use in other simulation environments using other technologies or nontechnological approaches. For example, a teacher educator or coach might use the materials for the basis of live role playing and adapt the interactor training materials to help preservice teachers play the role of students. The full set of project materials, including interactor training materials and guidelines for scoring the discussions, is archived in an online repository (https://data.qdr.syr.edu/dataverse/go-discuss) and is publicly available for use and adaptation.

References

Ball, D. L. (1993). Halves, pieces, and twoths: Constructing and using representational contexts in teaching fractions. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157–195). Lawrence Erlbaum Associates.

Ball, D. L., & Forzani, F. M. (2011). Building a common core for learning to teach: And connecting professional learning to practice. *American Educator, 35*(2), 17–38.

Barkai, R., Tsamir, P., Tirosh, D., & Dreyfus, T. (2002). Proving or refuting arithmetic claims: The case of elementary school teachers. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the Twenty-Sixth Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 57–64). University of East Anglia, School of Education and Professional Development.

Cazden, C. B. (1988). *Classroom discourse: The language of teaching and learning.* Heinemann.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5–8. https://doi.org/10.3102/0013189X032001005

Howell, H., & Mikeska, J. N. (2021). Approximations of practice as a framework for understanding authenticity in simulations of teaching. *Journal of Research on Technology in Education, 53*(1), 8–20. https://doi.org/10.1080/15391523.2020.1809033

Kazemi, E., & Stipek, D. (2009). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Journal of Education, 189*(1–2), 123–137. https://doi.org/10.1177/0022057409189001-209

Lamon, S. B. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Routledge.

Martin-Raugh, M., Reese, C. M., Howell, H., Tannenbaum, R. J., Steinberg, J. H., & Xu, J. (2016). *Investigating the relevance and importance of mathematical content knowledge areas for beginning elementary school teachers* (Research Memorandum No. RM-16-10). ETS.

Mikeska, J. N., Howell, H., & Straub, C. (2019). Using performance tasks within simulated environments to assess teachers’ ability to engage in coordinated, accumulated, and dynamic (CAD) competencies. *International Journal of Testing*, *19*(2), 128–147. https://doi.org/10.1080/15305058.2018.1551223

Mislevy, R. J., Steinberg, L. S., & Almond, R. G. (2002). On the structure of educational assessments. *Measurement: Interdisciplinary Research and Perspectives, 1*(1), 3–62. https://doi.org/10.1207/S15366359MEA0101_02

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*.

National Research Council. (2013). *Next Generation Science Standards: For states, by states*. The National Academies Press.

Newton, K. J. (2008). An extensive analysis of preservice elementary teachers' knowledge of fractions. *American Educational Research Journal, 45*(4), 1080–1110. https://doi.org/10.3102/0002831208320851

Reid, D., & Zack, V. (2009). Aspects of teaching proving in upper elementary school. In D. A. Stylianou, M. L. Blanton, & E. J. Kruth (Eds.), *Teaching and learning proof across the grades: A K–16 perspective* (pp. 133–146). Routledge.

Russell, S. J. , Schifter, D., Bastable, T., & Kasman, R. (2017). *But why does it work?: Mathematical argument in the elementary classroom.* Heinemann.

Stylianides, A. J., Bieda, K. N., & Morselli, F. (2016). Proof and argumentation in mathematics education research. In A. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 315–351). Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_9

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516–551. https://doi.org/10.3102/0034654308320292

Appendix: The Fractions Between Preservice Teacher–Facing Task Materials

ETS Research Study on Facilitating Student Discussions

The Generating Fractions Between Pairs of Fractions Discussion Task

Credit: Image courtesy of Mursion

Table of Contents

Note: The materials provided in the following sections are designed to help you plan for your discussion and should help you understand what you are supposed to do. You will also find teaching tips embedded throughout the document. An additional document, "An Introduction to the Simulated Classroom and Student Avatars," is also available for your use.

TEACHING TIP:
The teaching tips are designed to enhance your understanding of the task and your performance. You are **not** required to use them; instead they are here for you to use however you wish.

Introduction to the Generating Fractions Between Pairs of Fractions Discussion Task

What is the student learning goal for this discussion?

Student Learning Goal

Students will construct an argument about whether a method for generating a fraction in between two given fractions is worth remembering so that it can be used in the future.

What will you do?

Prior to the discussion, the students were introduced to a method that can be used for generating fractions in between two given fractions. This is called the "one-less" method and is described in detail in the Lesson Overview.

You will lead a discussion during which you should focus on providing students with opportunities to do the following.

TEACHING TIP: The method under consideration works sometimes but not always.

- **Present** their ideas to the class
- **Provide** reasoning or evidence to support their arguments
- **Listen** to and **respond** to one another's ideas and viewpoints
- **Consider** what makes a method worth remembering so that it can be used in the future

Your focus should be on engaging the students in discussion with one another and in the practice of argumentation. In order to achieve the learning goal, the students should interact with one another and think about the ideas of others. During the discussion, be sure to have students focus on providing reasoning or evidence to support their arguments and evaluating others' ideas and viewpoints. You should not introduce or discuss any other methods for generating fractions in between two given fractions.

TEACHING TIP: The student avatars may take a little time to think and respond. Provide wait time just as you would in a classroom.

You will have up to 20 minutes to lead this discussion in a simulated classroom environment made up of five upper-elementary student avatars. The students will be able to hear and see you, and they will respond in real time just like students in a real classroom. You can encourage the students to talk to one another, ask one another questions, and respond to one another's ideas.

Depending on how the discussion unfolds, you may or may not reach a satisfying conclusion by the end of the session time, and it is fine if you do not. If you do not reach a satisfying conclusion, just wrap up the discussion and indicate that you will pick the discussion up during the next class.

Lesson Overview

Student Learning Goal: Students will construct an argument about whether a method for generating a fraction in between two given fractions is worth remembering so that it can be used in the future.

> **TEACHING TIP:** Everything you do in planning for and leading this discussion should help the students make progress toward the student learning goal.

Background: The students are in fifth grade and are familiar with the following.

- Identifying parts of a fraction and using the vocabulary words "numerator" and "denominator"
- Writing fractions in standard notation (e.g., one half is $\frac{1}{2}$)
- Representing fractions using number lines and fraction bars, although students may not draw the number lines and fraction bars perfectly (e.g., with equal-sized wholes or equal-sized increments)
- Working with equivalent fractions
- Comparing fractions using a variety of methods, including using the fraction number-lines poster that is provided on the shared workspace

  Note: Not every student has the same level of understanding or ability with these content ideas and practices, but these are the learning opportunities that all students in this classroom have previously experienced.

> **TEACHING TIP:** You do not need to reteach or review these ideas. This information is provided so that you know where students are starting.

What the students have been doing and what they will be doing today:

This section provides a description of previous classwork so that you understand what the students already worked on. This will help you to prepare for the discussion at the point where you are dropped in. There are three parts.

1. What did the students do in the last class?
2. What have the students already done today?
3. What do you need to do in the discussion?

What did the students do in the last class?

In the last class, you asked the students to find one or two fractions between 2/3 and 7/8. One student, Emily, identified the fractions 3/4 and 4/5 using an unusual method that you thought might lead to a good discussion. You asked her to be ready to explain her method to the rest of the class during the next class meeting.

What have the students already done today?

1. Emily explained her method to the class.

   Review a short video of Emily explaining her method by selecting the image or using the following link: https://www.youtube.com/watch?v=FWbKrt5u19o&feature=youtu.be.

   Note: It is very important that you take the time to review the video so that you understand the method that Emily used. The discussion that the students will engage in is based on this method. The video is about 1 minute long.

I used a really easy way to find fractions between 2/3 and 7/8. I call it the "one-less" method.

I saw right away that the numerator in each fraction was one less than the denominator in each fraction.

What I mean is: In 2/3, 2 is one less than 3. And in 7/8, 7 is one less than 8.

So then I wrote down the other fractions like that, 3/4, 4/5, and 9/10. Then I checked them on the fraction number-lines poster to see if they were between 2/3 and 7/8.

I noticed that the ones that worked were the ones that had a denominator between 3 and 8. 2/3 and 7/8 are like the edges.

So then I realized that this is really fast because if you look at the denominators of all the one-less fractions, you can tell right off which ones are in between the edges and which ones aren't.

2. You assigned students to work together in the following groups.

   Group A: Mina and Will

   Group B: Carlos, Emily, and Jayla

3. Using only the method that Emily described, the groups were to think about how to find fractions between 2/3 and 7/8. You gave them a worksheet and asked them to write their answers to the following questions on the worksheet.

   a. Does this method work to find fractions in between 2/3 and 7/8? Explain.

   b. Would this method work for other pairs of fractions? Explain.

   c. Do you think this method should be kept in your toolbox? Make an argument to support your answer.

4. You reminded the students that they could use the fraction number-lines poster to determine whether certain fractions are between 2/3 and 7/8.
5. Each group answered the questions on the worksheet. The students have not shared their answers with the other group. The completed worksheets are available on the shared workspace.

What do you need to do in the discussion?

When the interaction begins, you will lead a discussion during which you should provide opportunities for the students to present their group work and engage in a discussion with one another.

Remember that the learning goal for this discussion is for the students to construct an argument about the use of the one-less method. Make sure that you provide opportunities for the students to do the following.

- **Present** their ideas to the class
- **Provide** reasoning or evidence to support their arguments
- **Listen** to and **respond** to one another's ideas and viewpoints
- **Consider** what makes a method worth remembering so that it can be used in the future

Remember, the groups have not shared their answers with one another yet, so one of the things you want to make sure happens in the discussion is that their answers get shared with the other group. By the end of the discussion, the students may not agree on whether the method is worth keeping in their toolbox; however, they should agree on when it can and cannot be used.

General Things to Notice About the Method

- The method Emily described *does work* for the given fractions (2/3 and 7/8). Noticing which denominators fall between 3 and 8 allows you to identify a set of fractions in between the given fractions.
- Emily did not explain *why* the method works, but there is a reason it works, and it relates to the sizes of the denominators and the sizes of the missing pieces in each fraction. The following is true when you make a list of all the one-less fractions and then consider it.
    - Every fraction is missing one piece from a whole (because they are one-less fractions).
    - Moving left to right, the denominators get larger (e.g., the 3 in 2/3 is larger than the 2 in 1/2).
    - This means the size of the missing piece gets smaller (e.g., thirds are smaller than halves, fourths are smaller than thirds).
    - So moving left to right, each fraction is missing a smaller piece than the one before, which means the fraction is larger than the one before.

1/2	**2/3**	**3/4**	**4/5**	**5/6**	**6/7**	**7/8**	**...**
Not between	Not between (lower boundary)	Between	Between	Between	Between	Not between (upper boundary)	Not between

- The method cannot be used for all pairs of fractions. For example, the method does not make sense if the given pair of fractions is 1/2 and 3/5, because the difference between the numerator and the denominator is not the same in both 1/2 and 3/5. 1/2 is a one-less fraction and 3/5 is not a one-less fraction.
- The method could be further generalized to situations in which the difference between the numerator and the denominator is the same in both fractions, but the differences are not 1. For example, this method would also work for fractions with a difference of 2 (e.g., 3/5 and 7/9) or fractions with a difference of 3 (e.g., 4/7 and 9/12).
- Sometimes, even if you have one-less fractions, the method will not give any results. For example, if the given fractions are 3/4 and 4/5, there are no fractions of this type between the boundaries, so no fractions can be found using the method.
- Even when the given fractions are one-less fractions, the method may produce fewer fractions than are requested. For example, the method gives only four fractions that are between 2/3 and 7/8, but fractions such as 5/7, 7/9, and 7/10 are also between 2/3 and 7/8.

- This method may be worth keeping in the toolbox for the following reasons.
  - It is quick and easy to use for very specific types of fraction pairs.
  - When it works, it can be faster than using other methods.
  - It is easy to remember if you take the time to look at the two "boundary fractions" and discover they are one-less fractions.
  - You can use it whenever the difference between the numerator and the denominator is the same in both fractions, not only when both fractions are one-less fractions. So you could use it to find fractions between 3/5 and 7/9 or between 4/7 and 9/12, for example.
- This method may **not** be worth keeping in the toolbox for the following reasons.
  - There are many pairs of fractions for which it cannot be used.
  - When it can be used, it provides a limited number of fractions that are between the given fractions, and you might be interested in finding more.

Student Responses: Mina and Will's Work

Name: Mina & Will

Does this method work to find fractions in between 2/3 and 7/8? Explain.

Sort of.
We found $\frac{5}{8}$ and $\frac{6}{7}$. We checked the fraction number lines poster. They worked.

It only gives you a few fractions and there are lots of fractions between $\frac{2}{3}$ and $\frac{7}{8}$.

Would this method work for other pairs of fractions? Explain.

No, it wouldn't work for $\frac{1}{2}$ and $\frac{2}{3}$ or $\frac{1}{2}$ and $\frac{3}{5}$.

Will you keep it in your toolbox?

No, because it doesn't work enough of the time.

Making Sense of Mina and Will's Work

Things to notice about Mina and Will's ideas about the method:

- The answer "sort of" is technically incorrect. The method does work to find fractions in between 2/3 and 7/8.
  - The question this group seems to be answering by writing "sort of" is "does the method give any fractions between 2/3 and 7/8?" This was not the question that was asked, but it is a reasonable question to consider when building their argument.
- They are correct that both 5/6 and 6/7 are between 2/3 and 7/8.
- They are also correct that there are many more fractions that are between 2/3 and 7/8 that the method does not generate. For example, 5/7 and 7/10 are between 2/3 and 7/8, but the method does not produce these fractions.
- They are correct that this method does not work for every pair of fractions.
  - The method does not work for their first example, 1/2 and 2/3, because there are no denominators between 2 and 3. The other group makes a similar statement when they write that the method "wouldn't work for two fractions if the denominators are next to each other."
  - The method does not work for their second example, 1/2 and 3/5, because the fractions are not both one-less fractions.
  - They did not explain why the method fails to work in each of these two examples.
- The evidence and reasoning that they use to support the argument against keeping the method in the toolbox is valid. There are significant limitations on the pairs of fractions for which the method can be used.

Student Responses: Carlos, Emily, and Jayla's Work

Name: Carlos, Emily, and Jayla

Does this method work to find fractions in between 2/3 and 7/8? Explain.

Yes!

$\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$ are all between $\frac{2}{3}$ and $\frac{7}{8}$.

Would this method work for other pairs of fractions? Explain.

It would give you a lot of fractions between $\frac{3}{4}$ and $\frac{89}{90}$. But it wouldn't work for two fractions if the denominators are next to each other.

Will you keep it in your toolbox?

Definately because it is quick and easy.

Making Sense of Carlos, Emily, and Jayla's Work

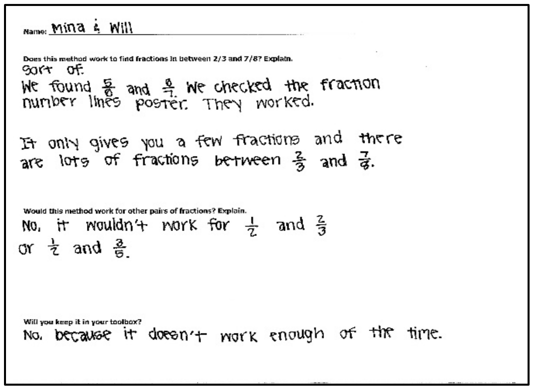
Things to notice about Carlos, Emily, and Jayla's ideas about the method:

- The group's observation that the method works for 2/3 and 7/8 is correct.
- They are correct that 3/4, 4/5, 5/6, and 6/7 are between 2/3 and 7/8.
- They are correct that this method would generate many fractions between 3/4 and 89/90 (85, to be exact).
    - By choosing a new pair of fractions that are both one-less fractions, they may be demonstrating that they realize the method would work for many other pairs of one-less fractions.
- They are also correct that this method would not work for two fractions if the denominators are next to each other. The other group makes a similar statement when they write that the method "wouldn't work for 1/2 and 2/3."
- The evidence and reasoning that they use to support their argument for keeping the method in the toolbox is valid. Methods can be useful without being applicable to many or all cases.

Shared Workspace Pages

The following are images of the shared workspace pages that will be available on the tablet. You and the students will be able to access and interact with these pages during the discussion. The tools on the toolbar can be used to draw or write on the pages. Blank pages are also available for you to use during the discussion.

Will and Mina's Work



Name: Mina & Will

Does this method work to find fractions in between 2/3 and 7/8? Explain.

Sort of.
We found $\frac{5}{6}$ and $\frac{6}{7}$. We checked the fraction number lines poster. They worked.

It only gives you a few fractions and there are lots of fractions between $\frac{2}{3}$ and $\frac{7}{8}$.

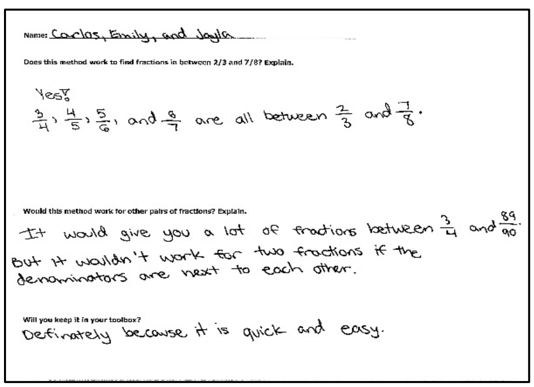
Would this method work for other pairs of fractions? Explain.

No, it wouldn't work for $\frac{1}{2}$ and $\frac{2}{3}$ or $\frac{1}{2}$ and $\frac{3}{5}$.

Will you keep it in your toolbox?

No, because it doesn't work enough of the time.

Carlos, Emily, and Jayla's Work



Name: Carlos, Emily, and Jayla

Does this method work to find fractions in between 2/3 and 7/8? Explain.

Yes!
$\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$ are all between $\frac{2}{3}$ and $\frac{7}{8}$.

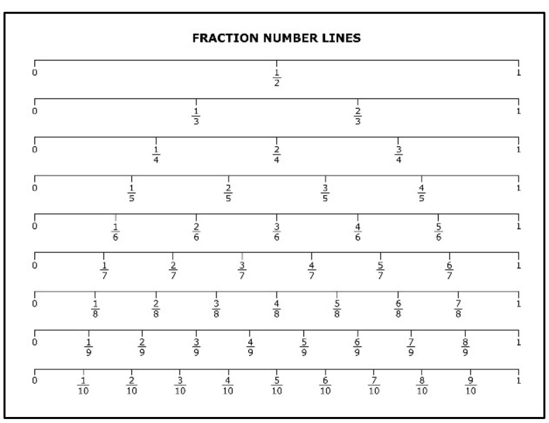
Would this method work for other pairs of fractions? Explain.

It would give you a lot of fractions between $\frac{3}{4}$ and $\frac{89}{90}$.
But it wouldn't work for two fractions if the denominators are next to each other.

Will you keep it in your toolbox?

Definately because it is quick and easy.

Fraction Number-Lines Poster



Features of High-Quality Discussions Focused on Argumentation

The discussion task you have been asked to complete is complex, and there are multiple approaches that you might take. The following list is a series of reflection questions for you to consider as you plan to lead a productive **discussion focused on engaging students in the practice of argumentation**. These features identify the main characteristics of high-quality discussions focused on argumentation. You might expect a helpful observer, such as a coach, peer, or instructor, to provide feedback on these features when observing your teaching in order to help you reflect on and learn from the experience.

1. **Attending to Students' Ideas:** Did I make sure every student's voice was heard and that all students' ideas were valued?
   - Did I give every student an opportunity to participate in meaningful ways?
   - Did I make sure to include all ideas that students shared in their previous work?
2. **Facilitating a Coherent and Connected Discussion:** Did the discussion make sense and feel organized and purposeful to the students?
   - Did I help the students make connections among ideas that build toward a shared understanding?
   - Did I help the students make sense of the discussion so that they could summarize the main takeaways and know what was learned?
3. **Encouraging Student-to-Student Interactions:** Did I succeed in getting students to engage in discussion with one another?
   - Did I encourage students to speak to one another directly?
   - Did I provide opportunities for students to pose questions to one another or comment on and critique one another's ideas?
4. **Developing Students' Conceptual Understanding:** Did I support students in developing a correct content understanding during the discussion?
   - Did I represent mathematics concepts correctly?
   - Did I give students opportunities to evaluate the correctness of content ideas so that they could learn how to be part of the process of critiquing those ideas?
   - Did I consider any mathematics content errors students had during the discussion and support students in working together to address those areas of confusion?
5. **Engaging Students in Argumentation:** Did the discussion allow students to engage in argumentation?
   - Did I focus the discussion on ideas that were worth debating?
   - Did I provide opportunities for the students to make claims or conjectures, support them with reasoning or evidence, and consider and critique their own and others' ideas?

TEACHING TIP:
Focus on getting the students to critique the method at hand, both the positive and the negative aspects. Try to avoid discussions about "what we did."

Video Examples of High-Quality Discussions Focused on Argumentation

Learning how to facilitate discussions focused on argumentation can be challenging. Observing examples of students and teachers engaged in these types of discussions can be helpful. The following video links will allow you to see what it looks like and sounds like when elementary and middle school students engage in productive argumentation in mathematics classrooms. We have also provided you with some questions to think about as you view these video examples and prepare to lead a productive **discussion focused on engaging students in the practice of argumentation**.

Focus	Things to notice	Resources (videos and vignettes)
Promoting student-to-student interaction	• How did the students engage in discussion with one another? • How did the teacher in each video promote student interaction? • What is the role of the teacher in each video and how are they different from one another?	Select the video titled "Strategy: Promoting Student Interaction In Science Seminars." Select the video "Joey's Run Part 1."
Supporting students' ownership of the ideas	• How does the teacher's focus on accurately capturing the students' meaning without changing it support their ownership of the ideas? • What does the teacher do to support a focus on students convincing one another?	Read the transcript of "Grade 4: Finding Equivalent Fractions."
Promoting students' evaluation and critique of competing claims	• How does the teacher use a focus on consensus to move the discussion forward? • How do the teaching moves focus on supporting students' critique?	Review the pdf and video of "Grade 3/4: Crazy Cakes."

Note. Resource examples are drawn from the Argumentation Toolkit website and from Illustrative Mathematics.

Notes

[1] The development of the simulated environment also included feedback from multiple stakeholders, including our advisory board, and a compilation of reviews and iterative refinements to the students' physical appearance as well as their voicing, background, and personality profiles. Although this development process happened concurrently, it is not described in this report.